Optimal Operation of Sanitary Pump Stations with Energy Storage During Power Outages

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Abstract—This paper aims to determine the optimal strategy to operate the pump in a sanitary pump station (SPS) to minimize its energy consumption over a specified time window. An SPS collects sanitary waste from nearby neighbourhoods in a city and then pumps the waste to a treatment plant for processing. Sustained operation of SPSs, even during power outages, is critical for the health and safety of urban dwellers. Energy storage can be used to operate the SPS when power from the grid is unavailable. Major challenges in minimizing energy consumption include (i) the nonlinear power characteristic of the pump, and (ii) the uncertainty in predicting the waste deposited into the SPS. We formulate a chance-constrained nonlinear optimal control problem and further devise a tractable solution strategy. We illustrate the energy savings obtained with the proposed control over the existing one.

Index Terms—Sanitary pump station, energy storage, chance constraints, resilience.

I. INTRODUCTION

Rapid urbanization along with more intense and frequent extreme weather events pose significant challenges for sustainable operation of critical city infrastructures, including water, sewer, and energy systems. As an example, the City of Richmond, located in British Columbia (BC), Canada, operates 152 sanitary pump stations (SPSs), and they ensure the collection of sanitary waste is pumped to the wastewater treatment plant for further processing [1]. Interruptions in SPS operation cause sanitary waste to accumulate at the station site, which poses serious health risks to residents of the city due to pathogens that can lead to spread of communicable diseases [2], [3]. Moreover, toxic pollutants in untreated waste may adversely affect ecosystems in nearby bodies of water [4]. Thus, essential to maintaining the health and well-being of city’s residents and their environment is uninterrupted operation of SPSs, but this may be compromised during a power outage. Indeed, recent natural disasters have caused severe damage to the distribution system and deprived critical infrastructures of electricity for prolonged periods of time [5], [6].

The City of Richmond has identified SPSs as critical infrastructures that require improved energy resilience in response to potential natural disasters like earthquakes and tsunamis. A compelling strategy to this end is to integrate local renewable energy sources and battery storage systems into SPS operation [7]. A secondary benefit of this approach is that, during normal operation when the SPS draws power directly from the grid, the energy from the renewable source can be sold back to the utility and the battery can be used to provide ancillary services to the bulk system [8]. These may be a useful source of income for the municipal government. As a starting point to achieve the above, in this paper, we focus on the scenario of operating an SPS in the City of Richmond with local battery storage in the event of a power outage. Particularly, given forecasts of sanitary-waste production, we seek to minimize the energy consumed by the SPS over a specified time window subject to operational constraints like waste-reservoir and pump-power characteristics.

Related work in the literature falls under the following categories: (i) sizing of and investment in distributed energy resources (DERs), and (ii) optimal operational control of critical-infrastructure systems. Sizing problems (see, e.g., [9], [10]) address the problem of estimating the optimal capacity of DERs like battery storage systems that satisfy the required load demand while minimizing capital and management costs. Sizing problems do not take into account operational management of infrastructures using DERs. In problems concerning optimal operational management of critical infrastructures (see, e.g., [11]–[15]), various control and optimization strategies are employed to minimize the energy consumed while keeping the system under consideration within certain constraints. However, existing literature in this domain does not explicitly address the operation of SPSs. The optimization problem addressed in this paper is similar to the one formulated in [14], which solves the problem of pump scheduling for a drainage system while minimizing the energy required to operate the system. In addition to the application scenario, our work differs from [14] in two ways. First, we consider the operation of constant-speed pumps, as they are already used in the City of Richmond, instead of variable-speed ones as in [14]. Moreover, our optimization problem formulation includes chance constraints to account for errors in predicted incoming flow rates of the sanitary waste.

The remainder of the paper is organized as follows. In Section II, we introduce notation and system models related to operating an SPS. Also, via a numerical example involv-
ing the SPS installed at Works Yard in Richmond BC, we motivate the need for improving pump operations to reduce energy consumption. In Section III, the proposed optimization problem is formulated and a companion solution approach follows. Numerical simulations involving the SPS at Works Yard is presented in Section IV to verify the proposed control strategy. Finally, concluding remarks and directions for future work are offered in Section V.

II. PRELIMINARIES

In this section, we introduce notation used throughout the paper and describe models for the sanitary pump station and its power consumption. We also outline the existing implementation of the pump controller and provide motivation for improving it.

A. Notation

The matrix transpose is denoted by $(\cdot)^T$. The spaces of $N \times 1$ real-valued vector are denoted by $\mathbb{R}^N$. The $N \times 1$ vectors with all zeros and ones are denoted by $\mathbf{0}_N$ and $\mathbf{1}_N$, respectively. For a vector $x := [x_1, \ldots, x_N]^T$, $x^2 := [x_1^2, \ldots, x_N^2]^T$.

B. Sanitary Pump Station

The sanitary pump station comprises a cylindrical reservoir, where the sanitary waste is collected. An electric motor pumps the waste out of the reservoir periodically to prevent it from overflowing. During normal operation, the battery is charged or discharged to provide ancillary service to the power system, e.g., frequency regulation [16]. When energy from the grid is unavailable due to, e.g., a power outage, the battery steps in to power the pump. We illustrate the aforementioned architecture and other pertinent modelling details in Fig. 1.

1) Reservoir Model: Since our goal is to synthesize controllers aided by measurements of sanitary-waste flows, we introduce the discrete-time index $k \in \mathbb{Z}_{\geq 0}$, which denotes the time $t_k = k\Delta t$ when the physical system is probed or actuated, where $\Delta t$ is the time elapsed between consecutive samples. Let $q_{in}(k)$ [m$^3$/s] and $q_{out}(k)$ [m$^3$/s] denote, respectively, the rates of sanitary waste flowing into and out from the reservoir at discrete time instant $k = 0, 1, \ldots$ Furthermore, let the $h(k)$ [m] denote the height dynamics of the sanitary waste collected inside the reservoir at time instant $k = 0, 1, \ldots$

Then, the height of the waste inside the reservoir can be approximated by the following:

$$h(k + 1) = h(k) + \frac{\Delta t}{a}(q_{in}(k) - q_{out}(k)), \quad h(0) = h_o, \quad (1)$$

where $a$ [m$^2$] denotes the area of the base of the reservoir and $h_o$ [m] is the initial height of the sanitary waste in the reservoir when the sanitary pump disconnects from the grid.

2) Pump Model: The electrical power consumed by the sanitary pump at time instant $k$, denoted by $p(k)$ [kW], is given by [17, Eq. (11)]

$$p(k) = \frac{\rho g \psi(k)}{\eta_{pump}} q_{out}(k), \quad (2)$$

where $\rho$ [kg/m$^3$] is the density of the sanitary waste, $g$ [m/s$^2$] is the gravitational acceleration, $\psi(k)$ [m] is the head gain generated by the pump, and $\eta_{pump}$ is efficiency characteristic of the pump and the efficiency of the pump is related to $q_{out}$ via a nonlinear relationship. In (2), the head gain is related to the outgoing flow rate by the following: [17, Eq. (1)]

$$\psi(k) = c_1 q_{out}^2(k) + c_2 q_{out}(k) \omega(k) + c_3 \omega^2(k), \quad (3)$$

where $c_1$, $c_2$, and $c_3$ are empirically determined parameters, and $\omega(k)$ is the ratio of actual to nominal pump speed at time step $k$. For a constant-speed pump,

$$\omega(k) \in \{0, \omega\}, \quad (4)$$

and for a variable-speed pump,

$$0 \leq \omega(k) \leq \omega. \quad (5)$$

3) Battery Storage Model: In the event of power outage, the sanitary pump would be operated with a battery storage unit of size $\bar{\sigma}$ [kWh]. The battery is discharged according to

$$e(k + 1) = e(k) - \frac{\Delta t}{\eta_{dis}} p(k), \quad e(0) = e_o, \quad (6)$$

where $p(k)$ is the power consumed by the pump as expressed in (2), $\eta_{dis}$ is the constant discharging efficiency of the battery, and $e_o$ [kWh] is the energy stored in the battery when the power outage occurs.

C. Problem Statement

The SPSs in the City of Richmond use constant-speed pumps, which operate at rated speed when the pump is on. In the existing implementation, the sanitary waste collects in the reservoir until the height of the waste rises above a certain threshold level $\bar{h}$ [m], at which point the pump attached to the reservoir turns on to dispose of the waste at constant flow rate $\bar{q}_{out} \in [0, \bar{q}_{out}]$, which corresponds to the most efficient operating point for the pump. The pump remains on until the height of the waste reaches a particular minimum level $\underline{h}$ [m], at which point the pump turns off, and the waste begins to accumulate in the reservoir again. In the event of power outage, the sanitary pump would be operated with a battery storage unit. Next, we illustrate the existing control strategy via
solve for the optimal sequence of \( q_{\text{out}}(k) \) that maximizes the time of operation while the SPS is powered by the battery, given predicted values of \( q_{\text{in}}(k) \).

### III. Optimal Control Synthesis

In this section, we formulate an optimal control problem to determine the sequence of \( q_{\text{out}}(k) \) that maximizes time of operation while the pump is powered by the battery system. Chance constraints are incorporated to account for uncertainty in \( q_{\text{in}}(k) \) forecasts. We further solve the proposed problem with a computationally tractable approach.

#### A. Problem Formulation

We assume that the rate of sanitary waste flowing into the reservoir at time \( k \) can be predicted based on, e.g., historical data, with some confidence. Hence, we model \( q_{\text{in}}(k) \), \( k = 1, \ldots, K - 1 \), as independent Gaussian random variables with mean \( \mu_q(k) \) and standard deviation \( \sigma_q(k) \), i.e., \( q_{\text{in}}(k) \sim \mathcal{N}(\mu_q(k), \sigma_q^2(k)) \).

1) **Cost Function:** To reflect the above, we minimize the energy used by the pump over a specified time window \( k = 0, \ldots, K - 1 \), as follows:

\[
\min_{q_{\text{out}}(k), \omega(k)} \Delta t \sum_{k=0}^{K-1} p(k) \Delta t, \tag{7}
\]

where \( p(k) \) is given by (2) if the pump is on with \( \omega(k) = \omega \), and \( p(k) = 0 \) if the pump is off with \( \omega(k) = 0 \).

2) **Reservoir Constraints:** The optimization problem is constrained by height dynamics of the sanitary waste in the reservoir given by (1). In (1), \( q_{\text{in}}(k) \) is an uncertain disturbance and \( q_{\text{out}}(k) \) is the control input to be optimized. The height of the sanitary waste must remain within some upper and lower bounds, \( \bar{h} \) and \( \underline{h} \), respectively. Since the sanitary-waste height is not a deterministic quantity due to uncertainty in \( q_{\text{in}}(k) \), we formulate the following chance constraints:

\[
\mathbb{P}(h(k) \leq \bar{h}) \geq 1 - \pi, \tag{8}
\]
\[
\mathbb{P}(h(k) \geq \underline{h}) \geq 1 - \pi, \tag{9}
\]

for each \( k = 1, \ldots, K \), requiring the probability of constraint violation to remain below an acceptable level \( \pi \in [0, 0.5] \).

3) **Pump Constraints:** In addition to the above, pump outgoing flow rates are constrained to

\[
0 \leq q_{\text{out}}(k) \leq \bar{q}_{\text{out}}, \tag{10}
\]

for each \( k = 0, \ldots, K - 1 \) which incidentally also constrains the discharge rate of the battery. Moreover, for a constant-speed pump, \( \omega(k) \in \{0, \omega\} \), for each \( k = 0, \ldots, K - 1 \).

#### B. Solution Approach

The problem outlined in Section III-A is challenging to solve due to several aspects:

(i) The decision variables \( \omega(k), k = 1, \ldots, K \), are discrete, and as a direct consequence, the cost function in (7) is discontinuous with respect to \( \omega(k) \).
Uncertainty in inflow predictions render the problem non-deterministic.

(iii) Although the heights are recursively solved based on current observations of inflows, chance constraints in (8) and (9) are enforced at each \( k = 1, \ldots, K \) over the entire specified time window.

Next, we address the aforementioned difficulties by reformulating the chance-constrained optimal control problem in Section III-A into a deterministic static optimization problem.

1) Reformulation of Cost Function: Using data provided in the pump specifications (see, e.g., [17, Fig. 3],[14, Eq. 8]), the power (equation (2)) consumed by the pump when it is on can be approximated by
\[
p(k) = p_{on}(k)(1 - e^{-\lambda q_{out}(k)}), \tag{11}
\]
where parameters \( \alpha, \beta, \gamma, \text{ and } \delta \) are obtained by curve fitting. The relationship in (11) incorporates nonlinear effects in head gain and pump efficiency. Physically, when the pump is on, it consumes some nominal power (i.e., \( \delta \) even when \( q_{out}(k) = 0 \). On the other hand, in this case, we may as well turn the pump off to conserve energy. In order to reflect the above intuition, we formulate the following relationship for the power consumed by the pump:
\[
p(k) = p_{on}(k)(1 - e^{-\lambda q_{out}(k)}), \tag{12}
\]
where \( \lambda \) is sufficiently large so that for some \( \epsilon_1 > 0 \), there exists \( \epsilon_2 > 0 \), such that \( e^{-\lambda q_{out}(k)} \leq \epsilon_1 \) for all \( q_{out}(k) > \epsilon_2 \).

In doing so, we use the continuous variable \( q_{out}(k) \) as a proxy for the discrete decision of the pump being on or off, i.e., the pump is off when \( q_{out}(k) = 0 \) and on otherwise. Also, by using (12) in (7), the cost function becomes continuous in \( q_{out}(k) \).

2) Reformulation of Chance Constraints: Since \( q_{in}(k) \), \( k = 0, \ldots, K - 1 \), are independent Gaussian random variables, the linear transformation in (1) implies that \( h(k) \), \( k = 1, \ldots, K \), are also normally distributed. Particularly, \( h(k) \sim N(\mu_h(k), \sigma^2_h(k)) \), where \( \mu_h(k) \) and \( \sigma_h(k) \) denote, respectively, the mean and standard deviation of the Gaussian random variable \( h(k) \). Based on the normality assumption, we reformulate the chance constraints in (8)–(9) into deterministic ones, as follows:
\[
\mu_h(k) + \sigma_h(k)\Phi^{-1}(1 - \pi) \leq \bar{H}, \tag{13}
\]
\[
\mu_h(k) - \sigma_h(k)\Phi^{-1}(1 - \pi) \geq \ll{h}, \tag{14}
\]
where \( \Phi^{-1}(\cdot) \) is the inverse of the cumulative distribution function [18].

3) Reformulation of Dynamics: Define vector variables
\[
\mu_Q := [\mu_q(0), \ldots, \mu_q(K - 1)]^T, \tag{15}
\]
\[
\sigma_Q := [\sigma_q(0), \ldots, \sigma_q(K - 1)]^T, \tag{16}
\]
\[
Q_{out} := [q_{out}(0), \ldots, q_{out}(K - 1)]^T. \tag{17}
\]
Further let
\[
\mu_H := [\mu_h(1), \ldots, \mu_h(K)]^T, \tag{18}
\]
\[
\sigma_H := [\sigma_h(1), \ldots, \sigma_h(K)]^T. \tag{19}
\]

Then, we evaluate entries of \( \mu_H \) and \( \sigma^2_H \) by repeatedly applying the linear transformation in (1) and stacking up the \( K \) instances of the resultant to get
\[
\mu_H = h_2\mathbbm{1}_K + A(\mu_Q - Q_{out}), \tag{20}
\]
\[
\sigma^2_H = B\sigma^2_Q, \tag{21}
\]
where
\[
A = \frac{\Delta t}{a} \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 1 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & 1 \end{bmatrix}, \quad B = \left( \frac{\Delta t}{a} \right)^2 \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 1 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & 1 \end{bmatrix}.
\]

C. Reformulated Problem

Based on the discussion above, we reformulate the problem in Section III-A into the following deterministic static optimization program:
\[
\min_{Q_{out}} \sum_{k=0}^{K-1} p_{on}(k)(1 - e^{-\lambda q_{out}(k)})\Delta t, \tag{22}
\]
subject to the constraints
\[
\mu_H = h_2\mathbbm{1}_K + A(\mu_Q - Q_{out}), \tag{23}
\]
\[
\sigma^2_H = B\sigma^2_Q, \tag{24}
\]
\[
\bar{h}\mathbbm{1}_K \geq h_K + \Phi^{-1}(1 - \pi)\sigma_H, \tag{25}
\]
\[
h_1^T \mathbbm{1}_K \leq H_K - \Phi^{-1}(1 - \pi)\sigma_H, \tag{26}
\]
\[
0 \leq Q_{out} \leq \mathbbm{1}_K. \tag{27}
\]

IV. CASE STUDY

In this section, we present a case study in which we apply the optimal control strategy described in Section III to reduce the consumption of power by the SPS at Works Yard, Richmond, BC. We solve the optimization problem (equations (22)–(27)) using the MATLAB function fmincon. We optimize the power consumption of the station between 5pm and 9pm during the month of September. Based on energy usage data provided by the City of Richmond, the average power consumption in this period is 6.080 kWh. We assume that the incoming flow rate \( q_{in}(k) \) belongs to a normal distribution with mean \( \mu_q(k) \) varying as shown in Fig. 3 and constant standard variation equal to 8% of the average of the profile in Fig. 3.

We compare the existing pump control scheme with the following strategies (all solving the optimization problem in (22)–(27)):

- **Case 1**: Optimization with actual incoming flow rates \( q_{in}(k) \) and no chance constraints (\( \sigma_Q = 0_K \) in (24)).
- **Case 2**: Optimization with predicted incoming flow rates \( \mu_q(k) \) and no chance constraints (\( \sigma_Q = 0_K \) in (24)).
- **Case 3**: Optimization with predicted flow rates \( \mu_q(k) \) and chance constraints (\( \pi = 0.05 \)).

In all the cases mentioned above, we solve the problem in (22)–(27) and compute the output flow rates every five minutes for the next five-minute interval. We compare the
power consumed by the pump as well as the number of violations of the waste-height constraints for the existing control scheme described in Example 1 and the three cases described above. Initial battery energy is taken to be 6.080 kWh for all scenarios. Energy remaining in the battery storage is plotted in Fig. 4, and the final battery energy and the number of height violations are reported in Table I.

From Table I, we observe that, under the ideal scenario where the incoming-flow forecast is perfectly known (i.e., Case 1), the outgoing flow rates obtained by solving the proposed optimization problem enforces the level of sanitary waste in the reservoir between the maximum and the minimum limits at all times. On the other hand, many violations of the height constraints occur in the existing scheme because the pump is turned on (off) after the waste level crosses the maximum (minimum) limits. The height constraints are violated in Case 2 and Case 3 because the optimization problem is not aware of the actual incoming flow rates. In Case 2, the problem in (22)–(27) assumes that the predicted incoming flow rates are perfectly known, while Case 3 accounts for the forecast uncertainty by including chance constraints. Thus, Case 3 has fewer constraint violations than Case 2. On the other hand, the pump consumes more energy in Case 3 than in Case 2, because the resulting outgoing flow rates are more conservative to account for forecast error. Finally, we note that pump indeed consumes less energy using the proposed optimization (in Case 1–Case 3) than under the existing control scheme.

<table>
<thead>
<tr>
<th>Height-constraint Violations</th>
<th>Existing</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
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<tr>
<td>Time [s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remaining Energy [kWh]</td>
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<td>2.2925</td>
<td>2.2949</td>
<td>2.2779</td>
</tr>
</tbody>
</table>

V. CONCLUDING REMARKS

We formulated a computationally tractable algorithm for operating an SPS that minimizes the energy consumed by the pump while maintaining the waste level in the reservoir within prescribed limits. In order to account for uncertainty in the incoming flow data, we incorporated chance constraints into the optimization problem, which reduced the number of height-constraint violations. We observed that the proposed control strategy enables the pump to consume less energy compared to the existing method. Important directions of further investigation would be to explore variable-speed pumps in the operation of SPSs, and to incorporate renewable sources to support the battery system while accounting for the uncertainty in renewable generation.

REFERENCES


