Projected grid-forming control for current-limiting of power converters

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Abstract—Sustainability and resilience concerns have motivated an unprecedented transformation of the electric power systems towards massive integration of renewable generation interfaced by power electronics. In this context, voltage source converters using grid-forming control are envisioned to provide services that so far have been provided by synchronous machines. In contrast to synchronous machines, voltage source converters are subject to stringent overcurrent limits. By exploiting the inherent time-scale separation between the inner control loops, the grid-forming reference dynamics (i.e., droop control, virtual oscillator control, etc.), and the transmission network, we highlight the well-known fact that the grid-forming reference dynamics need to be restricted to limit the converter output current without compromising stability. Next, we propose to formalize the problem of limiting the output current of a grid-forming converter by projecting the grid-forming reference dynamics onto a constraint on the output current. The main contribution is a projected droop controller that can be implemented using only local measurements. Moreover, we link the results to current limiting approaches using virtual impedance. Finally, we use a high-fidelity simulation to show that projected droop control outperforms virtual impedance current limiting.

I. INTRODUCTION

The electric power system is undergoing a period of unprecedented change. As a result of aiming towards more sustainable electric power systems, conventional thermal generation using synchronous machines is replaced by renewable energy sources that are connected to the system via power converters that do not contribute to stabilizing power systems [1], [2]. This results in larger, and more frequent, frequency deviations and jeopardizes the stability of today’s power system [2]. Moreover, compared to synchronous machines, power converters only have very limited overload capability [3], [4]. As a consequence, many transmission connected converters today further jeopardize grid stability by (momentarily) ceasing operation in conditions that may result in overcurrent [5].

Control strategies for grid-connected power converters can be broadly categorized [1] into (i) grid-forming strategies that regulate the voltage magnitude and frequency at the converter terminal to ensure self-synchronization and grid stability, similar to a synchronous machine, and (ii) grid-following controls that do not impose the voltage magnitude or frequency at their point of connection but follow an estimate of the frequency and voltage magnitude at the point of connection.

As a consequence, grid-forming power converters are envisioned to replace synchronous machines as the cornerstone of future power systems. The prevalent approach to grid-forming control is droop-control [6]–[9]. Other approaches include synchronous machine emulation [10], [11], virtual oscillator control that controls converters to mimic Liénard-type oscillators [12]–[14], and dispatchable virtual oscillator control [15], [16]. However, while grid-following control subject to current limits has been well studied [3], [17], the converter current limits are commonly neglected in the design and analysis of grid-forming control strategies.

Typically, grid-forming control is used as a reference model that governs the dynamics of the power converter through underlying cascaded voltage and current controllers [18]. Thus, the current can be limited by limiting the reference of the current controller [19]. However, this results in integrator windup [20] that typically causes a loss of synchronization or synchronous instability [21]. In contrast, virtual impedance current limiting does not rely on limiting the current reference, but uses the voltage controller to emulate a converter impedance that increases as the current approaches its limit [4], [22], [23]. While this heuristic implicitly limits the current, it is not obvious how to tune the virtual impedance, tuning guidelines are only available for special cases (i.e., symmetric short circuit faults), and the approach frequently fails for other scenarios (e.g., unbalanced faults).

In contrast, we propose to explicitly incorporate the current limit into the grid-forming controller by restricting its dynamics to a constraint set. To this end, we define a constraint that ensures that the quasi-steady-state current does not exceed its limits. Next, we propose to formalize the problem of limiting the output current of a grid-forming converter by projecting the grid-forming reference dynamics onto the quasi-steady-state current constraint. Motivated by this abstraction, we show how to approximately implement projected droop control using only local measurements. Moreover, we show that virtual impedance current limiting arises from minimizing a penalty function on the quasi-steady-state current and use this result to highlight the key differences between projected grid-forming control and virtual impedance current limiting. Finally, we use a high-fidelity case study to show that proposed projected droop control outperforms virtual impedance current limiting.
Notation

We use $\mathbb{R}$ and $\mathbb{N}$ to denote the set of real and natural numbers and define $\mathbb{R}_{\geq a} := \{ x \in \mathbb{R} | x \geq a \}$ and, e.g., $\mathbb{R}_{(a,b)} := \{ x \in \mathbb{R} | a \leq x < b \}$. Given $\theta \in [-\pi, \pi)$ the 2D rotation matrix is given by

$$\mathcal{R}(\theta) := \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \in \mathbb{R}^{2 \times 2}.$$  

Moreover, we define the $90^\circ$ rotation matrix $j := \mathcal{R}(\pi/2)$ that can be interpreted as an embedding of the complex imaginary unit $\sqrt{-1}$ into $\mathbb{R}^2$. Given a matrix $A$, $A^\top$ denotes its transpose. For column vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ we use $(x, y) = [x^\top, y^\top]^\top \in \mathbb{R}^{n+m}$ to denote a stacked vector and $\|x\|$ denotes the Euclidean norm. Furthermore, $I_n$ denotes the identity matrix of dimension $n$, and $\otimes$ denotes the Kronecker product. Matrices of size $n \times m$ are denoted by $0_{n \times m}$, and $0_n$ and $I_n$ denote column vector of zeros and ones of length $n$. Given a non-empty set $C \subseteq \mathbb{R}^n$ and vector $\mu \in \mathbb{R}^n$ we define the projection operator

$$[\mu]_C := \arg\min_{z \in C} \| \mu - z \|^2.$$  

II. CONVERTER AND POWER SYSTEM MODEL

A. Power Converter and device-level controls

The prevalent converter architecture for grid-forming DC/AC converters is the two-level voltage source converter shown in Figure 1 that consists of a DC source, a DC-link capacitor, a switching stage, and an output filter. Moreover, we assume that the converter is connected to the grid via a delta-wye transformer with grounded neutral point on the grid side. We consider a system of $N$ converters, and use $i_{f,k} \in \mathbb{R}^3$, $v_{f,k} \in \mathbb{R}^3$, $i_{o,k} \in \mathbb{R}^3$, and $v_{g,k} \in \mathbb{R}^3$ to denote the three-phase filter current, converter voltage, output current, and voltage at the connection point of the converter with index $k \in \mathcal{N} := \{1, \ldots, N\}$ in stationary abc coordinates. Furthermore, the modulation signal $m_{o,k} \in \mathbb{R}^3$ controls the modulated three-phase voltage $v_{sw,k} = \frac{1}{3}m_{o,k}v_{dc,k}$.  

While we use the detailed model shown in Figure 1 in the case studies in Section VI, we consider a simplified model for control design. To this end, we assume that the DC voltage $v_{dc,k}$ is regulated to be constant using $i_{dc,k}$ and $m_{o,k}$ is controlled by a current controller [18] that allows us to treat $i_{f,k}$ as a control input. Moreover, during unbalanced grid conditions (i.e., $\Pi^\top v_{g,k} \neq 0$), the delta-wye transformer prevents zero sequence current from reaching the converter (i.e., $\Pi^\top i_{o,k} = 0$) [26], [27]. Thus, we can simplify the analysis by applying the $dq$-transform (see e.g., [28, Ch. 6.1])

$$T(\theta_r) := \frac{2}{3} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin(\theta_r) & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$  

that removes the zero sequence component and transforms a three-phase signal into a coordinate frame rotating with the nominal grid frequency $\omega_0 \in \mathbb{R}_{>0}$, i.e., $\theta_r = \omega_0 t$.

Applying the change of coordinates results in $i_f := T(\theta_r) i_{f,k} \in \mathbb{R}^2$, $v_f := T(\theta_r) v_{f,k} \in \mathbb{R}^2$, $i_o := T(\theta_r) i_{o,k} \in \mathbb{R}^2$, and, considering the phase shift induced by the delta-wye transformer, the equivalent grid voltage as seen from the converter side of the transformer is given by $v_g := \mathcal{R}\left(\frac{\pi}{2}\right) T(\theta_r) v_{g,k}$.

Averaging the dynamics over one switching period results in the dynamics

$$C_{f,k} \frac{d}{dt} v_{f,k} = -Y_{f,k} v_{f,k} + i_{f,k} - i_{o,k},$$

$$L_{T,k} \frac{d}{dt} i_{o,k} = -Z_{T,k} i_{o,k} + v_{f,k} - v_{g,k},$$

where $Z_{T,k} := r_{T,k} L_2 + j\omega_0 L_2$, with $L_2 \in \mathbb{R}_{>0}$ and $r_{T,k} \in \mathbb{R}_{>0}$ models the transformer leakage impedance. Moreover, $c_{f,k} \in \mathbb{R}_{>0}$ denotes the filter capacitance and we define $\hat{C}_f := \frac{1}{2} c_{f,k}$ as well as the admittance matrix $Y_{f,k} := j\omega_0 c_{f,k}$. The resulting model is shown in Figure 2. Moreover, given a current set-point $i^*_f \in \mathbb{R}^2$ and limit

$$i_{max,k} \in \mathbb{R}_{>0},$$

we assume that the current is limited by

$$i_{f,k} = \begin{cases} i^*_f, & \| i^*_f \| \leq i_{max,k} \\ i_{sat,k} \frac{\| i^*_f \|}{\| i^*_f \|}, & \| i^*_f \| > i_{max,k} \end{cases}$$  

while preserving its directionality [20]. Commonly, the two-degree of freedom proportional-integral controller

$$\frac{d}{dt} v_{e,k} = v_{f,k} - v_{f,k},$$

$$i^*_{f,k} := i_{o,k} + Y_{f,k} v_{f,k} f_{f,k} = K_{p,k} (v^*_{f,k} - v_{f,k}) + K_{i,k} v_{e,k},$$

with gains $K_{p,k} \in \mathbb{R}_{>0}$ and $K_{i,k} \in \mathbb{R}_{>0}$ is used to tracks the voltage reference $v^*_{f,k} \in \mathbb{R}^2$ (see e.g. [18]) provided by a grid-forming controller (e.g., droop control).

B. Quasi-steady-state network model

For brevity of the presentation we assume that the voltage reference $v^*_{f,k}$ varies slowly enough such that a quasi-steady-state network model can be used [7], [16], [29], [30]. Let $g_{N,k} \in \mathbb{R}_{>0}$ and $b_{N,k} \in \mathbb{R}_{>0}$ denote the conductance and susceptance of the transmission line connecting node $k$ and node $l$. The quasi-steady-state network model is given by

$$i_{o,k} = \sum_{l=1}^{N} Y_{N,k,l} v_{g,k},$$

\(^1\)The controls developed in the remainder of the manuscript are independent of $\theta_r$ in abc coordinates, i.e., $\theta_r$ is not required to implement the controls.
where $Y_{N,kl} = g_{N,kl}I_2 + jb_{N,kl} \in \mathbb{R}^{2 \times 2}$ is the line admittance. Next, we can use the quasi-steady-state transformer model $Z_{T,k}i_{o,k}^* = v_f,k - v_{o,k}$ to eliminate the voltages $v_{o,k}$ and obtain the reduced quasi-steady-state network model

$$i_{o,k}^* = \sum_{l=1}^{N} Y_{kl}v_{f,l},$$

where $Y_{kl} = g_{kl}I_2 + jb_{kl} \in \mathbb{R}^{2 \times 2}$ denotes the corresponding line admittance of the reduced model.

## III. Control objectives

In this section we briefly review control objectives of grid-forming control of power converters and current limits of power converters. Based on these specifications, we propose to restrict the dynamics of the grid-forming control to a constraint that encodes the converter current limits.

### A. System-level control objectives

In this section we specify the system-level control objectives for a multi-converter power system. To this end we first define instantaneous active and reactive power and consistent power and voltage magnitude set-points.

**Definition 1 (Instantaneous Power)**

Given the voltage $v_{f,k}$ and the output current $i_{o,k}$, we define the instantaneous active power $P_k := v_{f,k}^Tv_{f,k} \in \mathbb{R}$ and the instantaneous reactive power $Q_k := v_{f,k}^TI_{o,k} \in \mathbb{R}$.

**Condition 1 (Consistent set-points)**

The set-points $P_k^* \in \mathbb{R}$, $P_k^* \in \mathbb{R}$, $V_k^* \in \mathbb{R}_{>0}$ are consistent with the network equations (6) if there exists $i_{o,k}$ and $v_{f,k}$ such that

$$i_{o,k} = \sum_{l=1}^{N} Y_{kl}v_{f,l}, \quad P_k^* = v_{f,k}^Tv_{f,k}, \quad Q_k^* = v_{f,k}^TI_{o,k},$$

and $\|v_{f,k}\| = V_k^*$ holds for all $(k,l) \in \{1, \ldots, N\} \times \{1, \ldots, N\}$.

The primary control objectives of grid-forming converter control is to ensure asymptotic stability of a nominal steady-state behavior specified by

$$\frac{df,v_{f,k}}{dt} = j\omega_0v_{f,k},$$

$$\{P_k, Q_k, \|v_{f,k}\|\} = \{P_k^*, Q_k^*, V_k^*\},$$

i.e., at the nominal steady-state, the voltage $v_{f,k}$ oscillates with the nominal frequency $\omega_0$, its magnitude is given by the setpoint $V_k^* \in \mathbb{R}_{>0}$, and the power injections $P_k$ and $Q_k$ match the set-points $P_k^*$ and $Q_k^*$.

### B. Grid-forming control and time-scale separation

Commonly a dynamic grid-forming controller of the form

$$\frac{dx_{c,k}}{dt} = f_k(x_{c,k}, u_{c,k}),$$

$$i_{o,k}^* = h_k(x_{c,k}),$$

with state $x_{c,k} \in \mathbb{R}^n$, input $u_{c,k} = (v_{f,k}, i_{o,k})$, and output $v_{f,k}^* = u_{o,k}^*$ is used to achieve the system-level objectives (7). Stability results for networks of converters with grid-forming control crucially rely on a separation of time-scales, i.e., that the controlled voltage $v_{f,k}$ converges to $v_{f,k}^*$ fast enough and that $v_{f,k}^*$ varies slowly enough such that the quasi-steady-state network model (6) can be used [8], [9], [16], [31].

While there is a wide body of literature on grid-forming control [6], [7], [10], [14], [16] the converter current limits are commonly neglected in the control design and analysis.

### C. Device-level current limits

Compared to synchronous machines power converters can only provide very limited overload current. In particular, each component of the three-phase current $i_{f,k}$ needs to be limited in order to avoid damaging the semiconductor switches. A straightforward approach to limit the converter current is to rely on the reference current limiter (4). However, when the current limitation (4) is active the voltage reference $v_{f,k}^*$ can generally no longer be tracked by the voltage controller (5). This breaks the assumption that $v_{f,k}$ converges to $v_{f,k}^*$, results in windup [20] of the integral control (5a), and leads to instability of standard grid-forming controls [21].

Loosely speaking, the voltage controller (5) can only track references when the current limit is not exceeded. Thus, the voltage references necessarily need to satisfy the following condition.

**Condition 2 (Quasi-steady-state feasibility)**

The voltage references $v_{f,k}^*$ are quasi-steady-state feasible if

$$\|Y_{f,k}v_{f,k}^* + \sum_{l=1}^{N} Y_{kl}v_{f,l}\| \leq i_{\text{max},k},$$

holds for all $k \in \{1, \ldots, N\}$.

Moreover, using (5b) and letting $i_{o,k} = i_{o,k}^*$ and $v_{f,k} = v_{f,k}^*$ we define the quasi-steady state filter current

$$i_{f,k}^* := i_{o,k} + Y_{f,k}v_{f,k}^*.$$}

Observe that $\|i_{f,k}^*\| \leq i_{\text{max},k}$ holds for quasi-steady-state feasible voltage references. In the remainder we assume that $v_{f,k}$ converges to $v_{f,k}^*$ if $v_{f,k}^*$ is quasi-steady-state feasible and that the current limiter (4) limits the current during transients of the voltage $v_{f,k}$ and the output current $i_{o,k}$.

### D. Projected grid-forming control

In practice, the desired response to events resulting in overcurrent (e.g. voltage sags, short circuit faults) is commonly specified using different case-by-case heuristics for different generation technologies [32]. While this ad-hoc approach may seem intuitive, it makes the overall system fault response complex and unpredictable.

In contrast, we propose to follow the grid-following dynamics as closely as possible while restricting them to the set

$$\mathcal{X}_k(t) := \{x_{c,k} \in \mathbb{R}^m \mid \|Y_{f,k}v_{f,k}^* + \sum_{l=1}^{N} Y_{kl}v_{f,l}\| \leq i_{\text{max},k}\}$$

of quasi-steady-state feasible voltage references. To formalize this approach, we require the following definition of a temporal tangent cone [24].

**Definition 2 (Temporal tangent cone)**

Consider a set-valued map $K : \mathbb{R} \rightharpoonup \mathbb{R}^n$. A vector $\mu \in \mathbb{R}^n$ is a temporal tangent vector of $K(t) \subseteq \mathbb{R}^n$ at a point $x \in K(t)$ and time $t$ if there exists sequences $x_k \to x$ and $\delta_k \to 0^+$.
such that \( x_k \in K(t+\delta_k) \) and \( x_k - x_k^{-\delta_k} \to \mu \). The set of all temporal tangent vectors of \( K \) at \( x \) is the temporal tangent cone denoted by \( T_x^K \). If \( K \) is time-invariant \( T_x^K \) denotes its tangent cone.

Figure 3 depicts two examples for tangent cones of a time-invariant set. Given a vector field \( f: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \), the associated time-varying projected dynamical system with state \( x \in \mathbb{R}^n \) and input \( u \in \mathbb{R}^m \) that is restricted to a (non-empty) time-varying constraint set \( K(t) \in \mathbb{R}^n \) is given by

\[
\frac{d}{dt} x = [f(x,u)]_{T_x^K}.
\]

In other words, if the solution of \( \frac{d}{dt} x = f(x,u) \) leaves the constraint set \( K \) at a point \( x \) and time \( t \) (i.e., point outside of the tangent cone \( T_x^K \)), the projection operator in (10) modifies the vector field such that it points inside the tangent cone \( T_x^K \). By definition of the tangent cone, this guarantees that the trajectory remains in the constraint \( K \) (for details please see [24]).

Conceptually, one could project the \( N \) grid-forming controllers (7) jointly onto the constraints \( X_1(t) \to X_N(t) \). However, this requires communication between the converters as well as full model knowledge and is not feasible in practice.

Thus, we treat the voltages \( v^*_f,d \) with \( l \neq k \) as exogenous signals resulting in the time-varying projected dynamics

\[
\frac{d}{dt} x_{c,k} = [f_k(x_{c,k},u_{c,k})]_{T_{x_{c,k}}^K},
\]

i.e., the projection defines a vector field that is compatible with the current constraint and matches the vector field of the original grid-forming dynamics as closely as possible.

Figure 3 shows the vector field of an unconstrained grid-forming control for a single converter connected to an infinite bus (see Figure 5). At the nominal grid voltage the unconstrained vector field at \( x_c \) points inside of the tangent cone of the current constraint \( X \). Consequently, the projection operator does not modify the vector field. In contrast, for a sag of the bus voltage the vector field at \( x_c \) is projected into the tangent cone \( T_{x_c,X} \) of \( X \) to limit the converter current.

In combination with a suitable grid-forming control strategy, the projected dynamics (11) define grid-forming controllers that explicitly incorporate current limits and minimizes the deviation from the corresponding unconstrained grid-forming control. However, it is not obvious how to implement the time-varying projected dynamics in practice (i.e., in discrete-time, using only local measurements, and with limited computational resources). Moreover, even though constraint violations can occur in practice, (11) does not specify dynamics outside of the constraint set. In the next section, we show how to address these concerns by approximately implementing a projected droop control.

IV. PROJECTED DROOP CONTROL USING FEEDBACK OPTIMIZATION

In this section we use a feedback optimization approach to design a projected droop controller that only requires local measurements.

A. Droop control

The most prevalent class of grid-forming control strategies is droop control [6]–[9], [31] which represents the reference voltage in polar coordinates, i.e., \( v^* \in \mathbb{R}^m, \theta_k \) denotes the droop voltage phase angle and \( V_k \) denotes the droop voltage magnitude. In its simplest form, droop control is defined by

\[
\begin{align}
\frac{d}{dt} \theta_k &= m_P(P^*_k - P_k), \\
V_k &= V^*_k + m_Q(Q^*_k - Q_k).
\end{align}
\]

Moreover, assuming that the network is lossless (i.e., \( g_{kl} = 0 \) for all \( k, l \in \{1, \ldots, N\} \)), the power injections are given by

\[
\begin{align}
P_k &= \sum_{l=1}^N b_{kl}V_kV_l \sin(\theta_k - \theta_l), \\
Q_k &= -\sum_{l=1}^N b_{kl}V_kV_l \cos(\theta_k - \theta_l).
\end{align}
\]

In the next section, we reverse engineer droop control as a gradient flow that minimizes an incremental energy function.

B. Reverse engineering droop control

Consider \( E := -\frac{1}{2} \sum_{k,l=1}^N b_{kl}V_kV_l \cos(\theta_k - \theta_l) \) and the incremental energy function \( \mathcal{U}: \mathbb{R}^{2N} \to \mathbb{R} \) inspired by [33]

\[
\mathcal{U} := E + \frac{1}{2} \sum_{k=1}^N \frac{1}{m_Q} \left| V_k - V^*_k \right|^2 - \theta_k P^*_k - (V^*_k)^{-1}V_k Q^*_k,
\]

where \( m_Q \in \mathbb{R}_{>0} \) denotes the reactive power droop coefficient. Next, observe that \( \nabla_{\theta_k} E = P_k \) and \( \nabla_V E = V^*_k \). Moreover, for small enough power set-points \( P^*_k \) and \( Q^*_k \)

![Fig. 3. Unconstrained grid-forming vector field (blue), current constraint \( X \) (orange) and tangent cone \( T_{x_c,X} \) (purple), and projected dynamics (green) for a converter during normal operation and a voltage sag.](image-url)
as well as sufficiently uniform voltage set-points \( V^\star_k \), the incremental energy function \( U \) has a minimum corresponding to \( (P_x, Q_k, V_k) = (P^\star_k, Q^\star_k, V^\star_k) \). Finally, the gradient-flow

\[
\frac{1}{m_P} \frac{d}{dt} \theta_k = -\nabla \theta_k U = -(P_k - P^\star_k),
\]

\[
\frac{\tau_v}{m_Q} \frac{d}{dt} \tilde{V}_k = -\nabla \tilde{V}_k U = \frac{1}{m_Q} ((V^*_k - \tilde{V}_k) - \left( \frac{1}{V_k} Q_k - \frac{1}{V^\star_k} Q^\star_k \right)),
\]

that minimizes \( U \) can be interpreted as a nonlinear droop controller with active and reactive power droop gains \( m_P, m_Q \in \mathbb{R}_{>0} \), and voltage control time constant \( \tau_v \in \mathbb{R}_{>0} \).

C. Projected droop control

For the approach proposed in this section it is crucial that constraint violations can be observed through measurements. However, if the current limiter (4) is active, we can not assume \( v_{f,k} = v^\star_{f,k} \). Therefore, we use the model

\[
\tilde{z}_{f,k} := Y_{f,k} v_{f,k} + Y_{kk} (v^\star_{f,k} - v_{f,k}) + i^\star_{o,k},
\]

which includes a correction term that accounts for the difference \( v^\star_{f,k} - v_{f,k} \) that arises if the current limiter (4) is active.

Next, we explicitly incorporate the current limit into droop control by combining the incremental energy function \( U \) with the constraint \( g_k := \|v^\star_{f,k} - v_{f,k}\|^2 - i^\star_{max,k} \leq 0 \) to obtain the minimization problem

\[
\min_{\theta_k, \tilde{V}_k} U \quad \text{subject to} \quad g_k \leq 0. \quad (14)
\]

Moreover, let \( \lambda_k \in \mathbb{R}_{>0} \) and \( \rho_k \in \mathbb{R}_{>0} \) denote a dual multiplier and positive penalty parameter, and consider the augmented Lagrangian \( L_k : \mathbb{R}^{2N} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R} \) given by

\[
L_k := U + \lambda_k g_k + \rho_k (\|g_k\|_{\mathbb{R}_{\geq 0}})^2.
\]

We propose to use the primal-dual dynamics (see e.g. [25])

\[
M \frac{d}{dt} x_{c,k} = -\nabla x_{c,k} L_k, \quad (15a)
\]

\[
\tau_x \frac{d}{dt} \lambda_k = [g_k(x_{c,k})]_{T_R k \geq 0}, \quad (15b)
\]

with \( M := \text{blkdiag}(\frac{1}{m_P}, \frac{1}{m_Q}) \) to track the time-varying optimizer of (14). The projection in (15b) restricts \( \lambda_k \) to \( \mathbb{R}_{>0} \), and \( \tau_x \in \mathbb{R}_{>0} \) is the time constant of the dual multiplier dynamics. Next, it can be verified that

\[
\nabla \lambda_k g_k = 2V_k [\sin(\theta_k) \cos(\theta_k)] (Y_{f,k} + Y_{kk}) T^\star_{f,k},
\]

\[
\nabla \tilde{V}_k g_k = 2 [\cos(\theta_k)] (Y_{f,k} + Y_{kk}) T^\star_{f,k}.
\]

Observe, that evaluating \( g_k \) and its gradient only requires local measurements and the admittance \( Y_{kk} \) that can be estimated from the grid short-circuit ratio and the transformer impedance. Overall we obtain

\[
\frac{1}{m_P} \frac{d}{dt} \theta_k = P_k - P^\star_k - \lambda_k \nabla \theta_k g_k - \rho_k [g_k(x_{c,k})]_{T_R k \geq 0} \nabla \theta_k g_k, \quad (16a)
\]

\[
\frac{\tau_v}{m_Q} \frac{d}{dt} \tilde{V}_k = \frac{1}{m_Q} (V^*_k - \tilde{V}_k) - \frac{1}{V_k} Q_k - \frac{1}{V^\star_k} Q^\star_k - \lambda_k \nabla \tilde{V}_k g_k - \rho_k [g_k(x_{c,k})]_{T_R k \geq 0}, \quad (16b)
\]

\[
\tau_x \frac{d}{dt} \lambda_k = [g_k(x_{c,k})]_{T_R k \geq 0}, \quad (16c)
\]

i.e. a nonlinear droop controller with added feedback that aims to enforce asymptotic constraint satisfaction.

\[\text{V. Threshold virtual impedance}\]

To limit the converter current while avoiding integrator wind up a heuristic has been proposed that does not rely on limiting the current reference (i.e., (4) is not active), but uses the voltage control loop (5) to emulate an increased filter impedance when the current \( i_{f,k} \) becomes too large [4], [22], [23]. In this section we show that this threshold virtual impedance can be recovered from an optimization problem that aims to minimize the violation of the current constraint.

To this end, we define the penalty function \( \Phi_k : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0} \)

\[
\Phi_k(\beta) :=
\begin{cases}
0 & \beta \leq i_{thr,k} \\
\beta^2 (\frac{1}{3} \beta - \frac{1}{2} i_{thr,k}) + \frac{1}{6} i_{thr,k}^3 & \beta > i_{thr,k}
\end{cases}
\]

where \( i_{thr,k} \in \mathbb{R}_{>0} \) is a threshold. The penalty function is depicted in Figure 4. It can be seen that \( \Phi_k(\beta) \) is convex, positive when \( \beta > i_{thr,k} \), and continuously differentiable.

![Fig. 4. Penalty function \( \Phi_k(\beta) \).](image)

Next, consider the unconstrained minimization problem

\[
\min_{v^\star_{f,k}} \|v^\star_{f,k} - v_{f,k}\|^2 + \rho_k \Phi_k(\|i_{f,k}\|), \quad (17)
\]

where \( v^\star_{f,k} \) denotes the voltage reference provided by the grid-forming control (8) and \( \rho_k \in \mathbb{R}_{>0} \) is a tuning parameter. The threshold virtual impedance is defined by [4], [22], [23]

\[
v^\star_{f,k} = v^\star_{f,k} - \rho_k (\|i_{f,k}\| - i_{thr,k})_{\mathbb{R}_{\geq 0}} Z_{V,k} v_{f,k}, \quad (18)
\]

where \( Z_{V,k} = r_{V,k} I_2 + j \omega_0 \ell_{V,k} \) and \( r_{V,k} \in \mathbb{R}_{>0} \) is a virtual resistance and \( \ell_{V,k} \in \mathbb{R}_{>0} \) is a virtual inductance. We require the following assumption.

\[\text{Assumption 1 (Output impedance)}\]

The transformer impedance \( Z_{T,k} \) dominates the output impedance, i.e., \( Y_{kk} = Z_{T,k}^{-1} \). Moreover, the filter capacitance is negligible, i.e., \( Y_{f,k} = 0 \).

The next result shows that (18) is the optimal solution to (17) if the voltage controller tracks its reference and the output current is in its quasi-steady-state.

\[\text{Theorem 1 (Threshold Virtual Impedance)}\]

Assume that \( v^\star_{f,k} = v^\star_{f,k}, \quad i^\star_{o,k} = i^\star_{o,k}, \) and \( i_{f,k} = i^\star_{f,k} \) holds. Under Assumption 1, the optimal solution to (17) is given by (18) with \( r_{V,k} = \frac{r_{\ell,k}}{r_{V,k} + i_{thr,k}^2} \) and \( \ell_{V,k} = \frac{r_{\ell,k}}{r_{V,k} + i_{thr,k}^2} \).

\[\text{Proof:}\] Note that, \( \Phi_k : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0} \) and \( \|v\|^2 \) are convex functions and that convexity is preserved by composition. It follows that (17) is a convex optimization problem. Moreover, for all \( 0 \leq \beta \leq i_{thr,k} \) it holds that \( \nabla_y \Phi_k(\beta) = 0 \) and for all \( \beta > i_{thr,k} \) it holds that \( \nabla_y \Phi_k(\beta) = \frac{\beta^2}{3} \).
\[ \beta (\beta - i_{\text{thr},k}). \] Next, using (5b), \( v_{f,k} = v_{f,k}^{\ast}, Y_{f,k} = 0, \]
\[ i_{f,k} = i_{f,k}^{\ast}, \quad \text{and} \quad i_{o,k} = i_{o,k}^{\ast} \]
it follows that \( \nabla v_{f,k} i_{f,k} = Y_{kk}^T. \)
Moreover, by Assumption 1 it holds that \( Y_{kk}^T = (Z_{T,k}^{-1}) = \frac{1}{r_{f,k}^2 + j\omega_{0} L_{f,k}} (r_{T,k} I_{2} + j\omega_{0} f_{T,k}). \)
Finally, letting \( \beta = \|i_{f,k}\| \)

Because the optimization (17) solely relies on a penalty function to encode the current constraint it is not obvious how to choose \( \rho_0 \) and \( i_{\text{thr},k} \) to ensure \( \|i_{f,k}\| \leq i_{\text{max},k} \)
tuning guidelines are only available for special cases (i.e., symmetric short circuit faults to ground). Moreover, the threshold virtual impedance (18) does not restrict the grid-forming dynamics to the current constraint, but decreases the constraint violation of its output. While this prevents windup of the voltage integrator (5a), integrator windup in the grid-forming control remains a concern. We emphasize that both of these concerns are not addressed by more complex virtual impedance current limiting strategies that utilize feed-forward control [4] and the hybrid strategy [34, Sec. III] that switches between limiting the current reference and threshold virtual impedance.

VI. CASE STUDIES

To illustrate the results and compare the projected droop control (16) to droop control (12) in combination with threshold virtual impedance (18), a detailed EMT simulation is used that contains an implementation of threshold virtual impedance current limiting, standard cascaded current and voltage control loops, and different grid-forming controls [35]. For the purpose of this study, the leakage impedance transformer model in [35] has been replaced by a linear three-phase two-winding delta-wye transformer model with the same leakage impedance.

In the remainder of this section we compare droop control with threshold virtual impedance current limiting (18) and the tuning from [35] with projected droop control (16). The droop coefficients \( m_P \) and \( m_Q \) are set to 2\% and the control gains of the current and voltage control are identical for both control strategies. The reference angle for the current and voltage loops is given by the droop control angle (i.e., \( \theta_{r,k} = \theta_{b} \)) for threshold virtual impedance and \( \theta_{r,k} = \omega_{0} t \) for projected droop control. Finally, for the projected droop controller (16), we assume that the transformer impedance \( Z_{T,k} \)
dominates the output impedance of the converter, i.e., \( Y_{kk} = Z_{T,k}^{-1} \). We emphasize that the current limiter (4) is not used in combination with virtual impedance current limiting.

A. Single-Converter Infinite-Bus

The first case study consists of a single converter connected to an infinite bus via two parallel transmission lines shown in Figure 5. We first simulate a symmetric three-phase short circuit fault to ground by closing the switch \( K_3 \)
that is cleared after 150 ms by disconnecting the line (i.e., opening \( K_1 \) and \( K_2 \)). The simulation results are shown in Figure 6, where \( \theta_{r,k} \) is the angle of the voltage reference \( v_{f}^{\ast} \) relative to a reference frame rotating with the nominal frequency \( \omega_{0} \). It can be seen that projected droop control results in a significantly smaller violation of the current limit, less oscillations in the voltage and current, and faster re-synchronization after the fault is cleared. In particular, projected droop control immediately reduces the reference voltage magnitude after the fault. In contrast, using threshold virtual impedance the voltage reference heavily oscillates before settling to a low enough magnitude. While both methods lose synchronization (i.e., deviate from the infinite bus frequency) during the fault\(^2\), projected droop control

\(^2\) This loss of synchronization is unavoidable because the converter is effectively disconnected from the infinite bus due to the fault.
adjusts its reference angle immediately after the fault is cleared and quickly recovers.

Next, we consider an unbalanced fault (phase A to B). The resulting converter current injection in stationary three-phase (i.e., abc) coordinates is shown in Figure 7. Observe that the combination of droop control with virtual impedance current limiting results in a severely distorted current injection that significantly exceeds the current limits of the converter. In contrast, projected droop control only results in a small and brief violation of the current limit.

![Fig. 7. Response of projected droop control (top) and droop control with virtual impedance current limiting (bottom) to an unbalanced fault (phase A to B). In contrast to the projected droop control, the virtual impedance current limiting results in a distorted current injection that exceeds the converter limits.](image)

**B. Multi-Converter System**

To evaluate projected droop control in a network of power converters, we use the three-bus system with 400 kV overhead lines and a 1125 MW constant impedance load shown in Figure 8. In this scenario, a sudden line opening between the smallest and largest converter is simulated by opening the switch $K_4$ at $t = 0.05$ s. Next, at $t = 0.55$ s, a symmetric short circuit fault to ground is applied by closing $K_3$ and the fault is cleared by disconnecting the line (i.e., opening $K_1$ and $K_2$). The results are depicted in Figure 9. The change in the network topology due to the line opening at $t = 0.05$ s changes the steady-state voltage phase angles. Observe that projected droop control adjusts the voltage reference angles $\angle v_{f,k}^*$ but not the voltage magnitude. In contrast, during the short circuit fault, the magnitude of the voltage reference is immediately decreased to limit the current. Moreover, once the fault is cleared the converters quickly re-synchronize. Note that the frequency does not return to the nominal frequency (i.e., the angles $\angle v_{f,k}^*$ do not return a constant value) because the power set-points are no longer consistent due to the changes in the network topology (see Definition 1).

![Fig. 8. Three-bus test case used to evaluate projected droop control in a multi-converter system.](image)

**VII. Conclusion**

In this work, we developed a grid-forming control strategy for power converters that explicitly considers the current limits of power electronic devices. In particular, we considered a standard control architecture comprising a cascaded...
current and voltage controller that tracks a reference provided by a grid-forming controller and proposed to restrict the dynamics of the grid-forming control to a constraint that encodes the converter current limits. The main contribution is a projected droop controller that explicitly accounts for the current limits of power converters and can be implemented using local measurements only. Moreover, we showed that virtual impedance current limiting arises from minimizing a penalty function on the quasi-steady-state current. Finally, we used a high-fidelity EMT simulation to illustrate the response of the proposed controller to grid faults and highlighted its improved performance compared to a standard virtual impedance current limiting strategy.

REFERENCES