Controller Synthesis for Multi-Agent Systems With Intermittent Communication: A Metric Temporal Logic Approach

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Abstract—This paper develops a controller synthesis approach for a multi-agent system (MAS) with intermittent communication. We adopt a leader-follower scheme, where a mobile leader with absolute position sensors switches among a set of followers without absolute position sensors to provide each follower with intermittent state information. We model the MAS as a switched system. The followers are to asymptotically reach a predetermined consensus state. To guarantee the stability of the switched system and the consensus of the followers, we derive maximum and minimal dwell-time conditions to constrain the intervals between consecutive time instants at which the leader should provide state information to the same follower. Furthermore, the leader needs to satisfy practical constraints such as charging its battery and staying in specific regions of interest. Both the maximum and minimum dwell-time conditions and these practical constraints can be expressed by metric temporal logic (MTL) specifications. We iteratively compute the optimal control inputs such that the leader satisfies the MTL specifications, while guaranteeing stability and consensus of the followers. We implement the proposed method on a case study with three mobile robots as the followers and one quadrotor as the leader.

I. INTRODUCTION

Coordination strategies for multi-agent systems (MAS) have been traditionally designed under the assumption that state feedback is continuously available. However, continuous communication over a network is often impractical, especially in mobile robot applications where shadowing and fading in the wireless communication can cause unreliability, and each agent has limited energy resources [1], [2].

Due to these constraints, there is a strong interest in developing MAS coordination methods that rely on intermittent information over a communication network. The results in [3]–[8] develop event-triggered and self-triggered controllers that utilize sampled data from networked agents only when triggered by conditions that ensure desired stability and performance properties. However, these results require a network represented by a strongly connected graph to enable agent coordination. This requirement of a strongly connected network induces constraints on the motion of the individual agents and additional maneuvers that may deviate from their primary purpose. Event-triggered and self-triggered control methods can also be used to coordinate the agents that communicate with a central base station or cloud intermittently as in [9], where submarines intermittently surface to obtain state information about themselves and their neighbors from a cloud. However, such a coordination strategy also requires additional maneuvers from the submarines that detract from their primary purpose.

Depending on the application and/or environment, some of the agents in a MAS may not be equipped with absolute position sensors. In such scenarios, the results in [3]–[8] are invalid. Therefore, there is a need for distributed methods capable of coordinating these agents that are not equipped with absolute position sensors while utilizing intermittent information. Moreover, such methods should not require agents to perform additional maneuvers to ensure the connectivity of the network. In [10], a set of followers operating with inaccurate position sensors are able to reach consensus at a desired state while a leader intermittently provides each follower with state information. By introducing a leader, the followers are able to perform their tasks without the need to perform additional maneuvers to obtain state information.

Building on the work of [10], we adopt a leader-follower scheme, where the MAS is modeled as a switched system [11], [12]. As an illustrative example shown in Fig. 1, the three followers need to reach consensus at the center of the green feedback region and one leader agent is to provide intermittent state information to each follower. To guarantee the stability of the switched system and consensus of the followers, we derive maximum and minimal dwell-time conditions to constrain the intervals between consecutive time instants at which the leader should provide state information to the same follower.

The maximum and minimum dwell-time conditions can be encoded by metric temporal logic (MTL) specifications [13]. Such specifications have also been used in many robotic applications for time-related specifications [14], [15]. Furthermore, as the leader is typically more energy-consuming and safety-critical due to the high-quality sensing, communication and mobility equipments, the leader is likely required to satisfy additional MTL specifications for practical constraints such as charging its battery and staying in specific regions. In the example shown in Fig. 1, the leader needs to satisfy an MTL specification “reach the charging station \(G_1\) or \(G_2\) in every 6 time units and always stay in the yellow region \(D\)”.

We design the followers’ controllers such that guarantees on the stability of the switched system and consensus of the followers hold, provided that the maximum and minimal
dwell-time conditions are satisfied. Then we synthesize the leader’s controller to satisfy the same MTL specifications that encode the maximum and minimal dwell-time conditions and the additional practical constraints. There is a rich literature on controller synthesis subject to temporal logic specifications [16]–[26]. For linear or switched linear systems, the controller synthesis problem can be converted into a mixed-integer linear programming (MILP) problem [18], [19]. Additionally, as the followers are not equipped with absolute position sensors, we design an observer to estimate the followers’ states and the state estimates can change abruptly due to the intermittent communication of state information. Therefore, we solve the MILP problem iteratively to account for such abrupt changes.

We provide an implementation of the proposed method on a simulation case study with three mobile robots as the followers and one quadrotor as the leader. The results in two different scenarios show that the synthesized controller can lead to satisfaction of the MTL specifications, while achieving the stability of the switched system and consensus of the followers.

II. BACKGROUND AND PROBLEM FORMULATION

A. Agent Dynamics

Consider a multi-agent system (MAS) consisting of $Q$ followers ($Q \in \mathbb{Z}_{>0}$) index by $i \in F \triangleq \{1,\ldots,Q\}$ and a leader indexed by 0. Let the time set be $T = \mathbb{R}_{\geq 0}$. Let $y_0$, $y_i : T \rightarrow \mathbb{R}^2$ denote the position of the leader and follower $i$, respectively. Let $x_0 : T \rightarrow \mathbb{R}^l$ and $x_i : T \rightarrow \mathbb{R}^m$ denote the state of the leader and follower $i$, respectively. The linear time-invariant dynamics of the leader and follower $i$ are

$$
\begin{align*}
\dot{x}_0 (t) &= A_0 x_0 (t) + B_0 u_0 (t), \\
y_0 (t) &= C_0 x_0 (t), \\
\dot{x}_i (t) &= A_\zeta i (t) + B u_i (t) + d_i (t), \\
y_i (t) &= C x_i (t),
\end{align*}
$$

where $A_0 \in \mathbb{R}^{l \times l}, A_\zeta i \in \mathbb{R}^{m \times m}, B_0 \in \mathbb{R}^{l \times n}, B \in \mathbb{R}^{m \times n}$, $C_0 \in \mathbb{R}^{2 \times l}, C \in \mathbb{R}^{2 \times m}$. Here, $u_0$, $u_i : T \rightarrow \mathbb{R}^n$ denote the control inputs of the leader and follower $i$, respectively, and $d_i : T \rightarrow \mathbb{R}^m$ is an exogenous disturbance. For simplicity, we assume that $\lambda_{\text{max}} (A)^2 \in \mathbb{R}_{>0}$ and $B$ has full row rank.

B. Sensing and Communication

Each follower is equipped with a relative position sensor and hardware to enable communication with the leader. Since the followers lack absolute position sensors, they are not able to localize themselves within the global coordinate system. Nevertheless, the followers can use their relative position sensors to enable self-localization relative to their initially known locations. However, relative position sensors like encoders and inertial measurement units (IMUs) can produce unreliable position information since e.g., wheels of mobile robots may slip and IMUs may generate noisy data. Hence, the $d_i (t)$ term in (1) models the inaccurate position measurements from the relative position sensor of follower $i$ as well as any external influences from the environment. Navigation through the use of a relative position sensor results in dead-reckoning, which becomes increasingly more inaccurate with time if not corrected. On the other hand, the leader is equipped with an absolute position sensor and hardware to enable communication with each follower. Unlike a relative position sensor, an absolute position sensor allows localization of the agents within the global coordinate system.

The followers’ task is to reach consensus to a predetermined state $x_\zeta \in \mathbb{R}^m$. A feedback region (see Fig. 1) centered at the position $C x_\zeta \in \mathbb{R}^2$ with radius $R_g \in \mathbb{R}_{>0}$ is capable of providing state information to each follower $i \in F$ once $\|y_i (t) - C x_\zeta\| = \|C x_i (t) - C x_\zeta\| \leq R_g$. The leader’s task is to provide state information to each follower while they navigate to $x_\zeta$ with the intermittent state information. Both the leader and the followers are equipped with digital communication hardware where communication is only possible at discrete time instants. Let $R_c \in \mathbb{R}_{>0}$ and $R_s \in \mathbb{R}_{>0}$ denote the communication and sensing radii of each agent, respectively. For simplicity, let $R_c = R_s \triangleq R$.

The leader provides state information to the follower $i$ (i.e., services the follower $i$) if and only if $\|y_i (t) - y_0 (t)\| \leq R$ and the communication channel of the follower $i$ is on. We define the communication switching signal $\zeta_i$ for follower $i$ as $\zeta_i = 1$ if the communication channel is on for follower $i$, and $\zeta_i = 0$ if the communication channel is off for follower $i$. We use $t^i_{s+1} \geq 0$ to indicate the $s^{th}$ servicing time instance for follower $i$. Hence, the $(s+1)^{th}$ servicing time instant for follower $i$ is

$$
t^i_{s+1} \triangleq \inf \{ t \geq t^i_s : (\|y_i (t) - y_0 (t)\| \leq R) \land (\zeta_i (t) = 1) \}
$$

where $\land$ denotes the conjunction logical connective.

C. State Observer and Error Dynamics

The followers, not equipped with absolute position sensors, implement the following model-based observer to esti-

[^2]: $\lambda_{\text{max}} (A)$ denotes the maximum singular value of $A$.

[^3]: For $s = 0$, $t^i_0$ is the initial time, for simplicity we take $t^i_0 = 0$. 

Fig. 1: Illustrative example of a MAS with a leader (quadrotor) and three followers (mobile robots).
mate the state of each follower \( i \in F \):
\[
\dot{x}_i(t) = A\dot{x}_i(t) + Bu_i(t), \quad t \in [t^i_s, t^{i+1}_s),
\]
where \( \dot{x}_i : \mathbb{T} \rightarrow \mathbb{R}^m \) denotes the estimate of \( x_i \).

Then we can obtain the position estimate of follower \( i \) as
\[
y_i(t) \triangleq C\dot{x}_i(t).
\]

To facilitate the analysis, we define the following two error signals
\[
e_{1,i}(t) \triangleq \dot{x}_i(t) - x_i(t)
\]
and
\[
e_{2,i}(t) \triangleq x_g - \dot{x}_i(t).
\]

Similar to [10], we adopt the following assumptions.

**Assumption 1:** The state estimate \( \dot{x}_i \) is initialized as \( \dot{x}_i(0) = x_i(0) \) for all \( i \in F \).

**Assumption 2:** The leader has full knowledge of its own state \( x_0(t) \) for all \( t \geq 0 \) and the initial state \( x_i(0) \) for all \( i \in F \).

**Assumption 3:** The disturbance \( d_i \) is bounded, i.e., \( |d_i(t)| \leq \overline{d}_i \) for all \( t \geq 0 \), where \( \overline{d}_i \in \mathbb{R}_{>0} \) is a known constant.

The control of follower \( i \) is as follows:
\[
u_i(t) \triangleq -B^+ A\dot{x}_i(t) + k_i B^+ e_{2,i}(t)
\]
such that \( B^+ \) denotes the pseudo-inverse of \( B \) and \( k_i \in \mathbb{R}_{>0} \) is a user-defined parameter. Since \( B \) has full row rank (see Section II-A), \( B B^+ = I_{m \times m} \), where \( I_{m \times m} \) is the identity matrix.

At each servicing time instant \( t^i_s \), with the feedback provided by the leader, the state estimate \( \dot{x}_i \) of follower \( i \) immediately resets to \( x_i \). Therefore, the state estimates follow the dynamics of switched systems [27].

Substituting (1) and (2) into the time-derivative of (4) yields
\[
\dot{e}_{1,i}(t) = Ae_{1,i}(t) - d_i(t), \quad t \in [t^i_s, t^{i+1}_s),
\]
\[
e_{1,i}(t^i_s) = 0_m,
\]
where \( 0_m \in \mathbb{R}^m \) is the zero column vector. Substituting (2) into the time-derivative of (5) yields
\[
\dot{e}_{2,i}(t) = -k_i e_{2,i}(t), \quad t \in [t^i_s, t^{i+1}_s),
\]
\[
e_{2,i}(t^i_s) = x_g - x_i(t^i_s).
\]

**D. Metric Temporal Logic (MTL)**

To achieve the stability of the switched system and consensus of the followers while satisfying the practical constraints of the leader, the requirements of the MAS can be specified in MTL specifications (see details in Section IV). In this subsection, we briefly review the MTL interpreted over discrete-time trajectories [28]. The domain of the position of the agents \( y \) is denoted by \( \mathcal{Y} \subset \mathbb{R}^z \). The domain \( \mathbb{B} = \{\text{true, false}\} \) is the Boolean domain, and the time index set is \( \mathbb{I} = \{0, 1, \ldots\} \). With slight abuse of notation, we use \( y \) to denote the discrete-time trajectory as a function from \( \mathbb{I} \) to \( \mathcal{Y} \). A set \( AP \) is a set of atomic propositions, each mapping \( \mathcal{Y} \) to \( \mathbb{B} \). The syntax of MTL is defined recursively as follows:
\[
\phi \triangleq T \mid \pi \mid \neg \phi \mid \phi_1 \land \phi_2 \mid \phi_1 \lor \phi_2 \mid \phi_1 \Upsilon \phi_2
\]
where \( T \) stands for the Boolean constant True, \( \pi \in AP \) is an atomic proposition, \( \neg \) (negation), \( \land \) (conjunction), \( \lor \) (disjunction) are standard Boolean connectives, \( \Upsilon \) is a temporal operator representing “until” and \( \tau \) is a time interval of the form \( \tau = [j_1, j_2) \) \((j_1 \leq j_2, j_1, j_2 \in \mathbb{I}) \). We can also derive two useful temporal operators from “until” \( \Upsilon (\tau) \), which are “eventually” \( \Upsilon \tau \phi = \Upsilon \Upsilon \tau \phi \) and “always” \( \Upsilon \tau \phi = \neg \Upsilon \tau \neg \phi \). We define the set of states that satisfy the atomic proposition \( \pi \) as \( \mathcal{O}(\pi) \in \mathcal{Y} \).

Next, we introduce the Boolean semantics of MTL for trajectories of finite length in the strong and the weak view, which are modified from the literature of temporal logic model checking and monitoring [29]–[31]. We use \( t[j] \in \mathbb{T} \) to denote the time instant at time index \( j \in \mathbb{I} \) and \( y^j \triangleq y(t[j]) \) to denote the value of \( y \) at time \( t[j] \).

In the following, \( (y^{0:H}, j) \models AP \) (resp. \( (y^{0:H}, j) \models \neg AP \)) means the trajectory \( y^{0:H} \models y^{0} \ldots y^{H} (H \in \mathbb{Z}_{>0}) \) (resp. weakly) satisfies \( AP \) at time index \( j \), \( (y^{0:H}, j) \not\models AP \) (resp. \( (y^{0:H}, j) \not\models \neg AP \)) means \( y^{0:H} \) fails to strongly (resp. weakly) satisfy \( AP \) at time index \( j \).

**Definition 1:** The Boolean semantics of MTL for trajectories of finite length in the strong view is defined recursively as follows [23]:
\[
(y^{0:H}, j) \models S \pi \Leftrightarrow j \leq H \land y^j \in \mathcal{O}(\pi),
\]
\[
(y^{0:H}, j) \models S \neg \pi \Leftrightarrow (y^{0:H}, j) \not\models \neg \pi,
\]
\[
(y^{0:H}, j) \models S \pi \land \pi \Leftrightarrow (y^{0:H}, j) \models \pi,
\]
\[
(y^{0:H}, j) \models S \pi \lor \pi \Leftrightarrow (y^{0:H}, j) \models \pi,
\]
\[
(y^{0:H}, j) \models S \pi \Upsilon \phi \Leftrightarrow \exists j' \in j + \mathbb{I}, s.t. (y^{0:H}, j') \models S \phi_2,
\]
\[
(y^{0:H}, j) \not\models S \pi \Upsilon \phi \Leftrightarrow \forall j' \in [j, j'], (y^{0:H}, j') \models \neg S \phi_2.
\]

**Definition 2:** The Boolean semantics of MTL for trajectories of finite length in the weak view is defined recursively as follows [23]:
\[
(y^{0:H}, j) \models W \pi \Leftrightarrow (y^{0:H}, j) \models \pi \text{ or either of the following holds:}
\]
\[
1) \quad j \leq H \land y^j \in \mathcal{O}(\pi);
\]
\[
2) \quad j > H,
\]
\[
(y^{0:H}, j) \models W \neg \pi \Leftrightarrow (y^{0:H}, j) \not\models \neg \pi,
\]
\[
(y^{0:H}, j) \models W \pi \lor \pi \Leftrightarrow (y^{0:H}, j) \models \pi,
\]
\[
(y^{0:H}, j) \models W \pi \Upsilon \phi \Leftrightarrow \exists j' \in j + \mathbb{I}, s.t. (y^{0:H}, j') \models W \phi_2,
\]
\[
(y^{0:H}, j) \not\models W \pi \Upsilon \phi \Leftrightarrow \forall j' \in [j, j'), (y^{0:H}, j') \models \neg W \phi_2.
\]

Intuitively, if a trajectory of finite length can be extended to infinite length, then the strong view indicates that the truth value of the formula on the infinite-length trajectory is already “determined” on the trajectory of finite length, while the weak view indicates that it may not be “determined” yet [31]. As an example, a trajectory \( y^{0:3} = y^0, y^1, y^2, y^3 \) is not possible to strongly satisfy \( \phi = [0, 5] \pi \) at time 0, but \( y^{0:3} \) is
possible to strongly violate $\phi$ at time $0$, i.e., $(y_1^{1:3}, 0) \models \neg \phi$ is possible.

For an MTL formula $\phi$, the necessary length $\|\phi\|$ is defined recursively as follows [32]:

$$
\|\pi\| = 0, \quad \|-\phi\| = \|\phi\|,
\|\phi_1 \land \phi_2\| = \max(\|\phi_1\|, \|\phi_2\|),
\|\phi_1 U_{[j_1, j_2]} \phi_2\| = \max(\|\phi_1\|, \|\phi_2\|) + j_2.
$$

E. Problem Statement

We now present the problem formulation for the control of the MAS with intermittent communication and MTL specifications.

Problem 1: Design the control inputs for the leader $u_0 = [u_0^1, u_0^2, \ldots]$, such that the following characteristics are satisfied while minimizing the control effort $\|u_0\|^4$:

Correctness: A given MTL specification $\phi$ is weakly satisfied by the trajectory of the leader.

Stability: The error signal $e_{1,i}(t)$ is uniformly bounded, and the error signal $e_{2,i}(t)$ is asymptotically regulated\(^5\) for each follower $i$.

Consensus: The states of the followers asymptotically reach consensus to $x_g$.

III. Stability and Consensus Analysis

In this section, we provide the conditions for achieving the stability of the switched system and the consensus of the followers. Such conditions include maximal (see Theorem 1) and minimal (see Theorem 2) dwell-time conditions on the intervals between consecutive time instants at which the leader should provide state information to the same follower.

Theorem 1: Let $V_T \in \mathbb{R}_{>0}$ be a user-defined parameter. Then, the error signal in (4) for follower $i$ is uniformly bounded, i.e., $\|e_{1,i}(t)\| \leq V_T$ for all $t \geq 0$, provided the leader satisfies the maximum dwell-time condition

$$
t_{i,s+1} - t_i^s \leq \frac{1}{\lambda_{\max}(A)} \ln \left( \frac{\lambda_{\max}(A) V_T}{\delta_i} + 1 \right)
$$

for all $s \in \mathbb{Z}_{\geq 0}$.

Proof: The proof was omitted due to lack of space here. Please refer to the extended version [33].

Theorem 2: The error signal in (5) is globally asymptotically regulated provided the leader satisfies both the maximum dwell-time condition in (11) and the minimum dwell-time condition in

$$
t_{i,s+1} - t_i^s \geq \frac{1}{k_i} \ln \left( \frac{\|e_{2,i}(t_i^s)\|}{\|e_{2,i}(t_i^s)\| - V_T} \right)
$$

for all $s \in \mathbb{Z}_{\geq 0}$ such that $s < s$ (s denotes the index of $t_i^s$ where $\|e_{2,i}(t_i^s)\| \leq V_T$ first holds), and $V_T \in \left(0, \frac{R_g}{2 \lambda_{\max}(C)}\right]$.

\(^4\|\|\|$ denotes the 2-norm.

\(^5\)The error signal $e_{2,i}(t)$ is asymptotically regulated if $\|e_{2,i}(t)\| \to 0$ as $t \to \infty$.

Proof: Suppose the leader satisfies the dwell-time condition in (11) for all $s \in \mathbb{Z}_{\geq 0}$. Consider the common Lyapunov functional $V_{2,i} : \mathbb{R}^m \to \mathbb{R}_{\geq 0}$

$$
V_{2,i}(e_{2,i}(t)) = \frac{1}{2} e_{2,i}^T(t) e_{2,i}(t).
$$

By (1) and (2), (4) is continuously differentiable over $[t_i^s, t_i^{s+1}]$. Substituting (9) when $t \in [t_i^s, t_i^{s+1}]$ into the time-derivative of (13) yields

$$
\dot{V}_{2,i}(e_{2,i}(t)) = -k_i e_{2,i}^T(t) e_{2,i}(t)
$$

where substituting (13) into (14) yields

$$
\dot{V}_{2,i}(e_{2,i}(t)) = -k_i V_{2,i}(e_{2,i}(t)).
$$

The solution of (15) over $[t_i^s, t_i^{s+1}]$ is given by

$$
V_{2,i}(e_{2,i}(t)) = V_{2,i}(e_{2,i}(t_i^s)) e^{-k_i(t_i^s-t_i^s)}
$$

where substituting (13) results in

$$
\|e_{2,i}(t)\| \leq \|e_{2,i}(t_i^s)\| e^{-k_i(t_i^s-t_i^s)}.
$$

Observe that $e_{2,i}(t_i^s)$ is finite since $x_i(t_i^s)$ is a measured quantity provided by the leader where (16) implies $e_{2,i}(t)$ is bounded over $[t_i^s, t_i^{s+1}]$. Moreover, the RHS of follower $i$'s dynamics in (1) are Lebesgue measurable and locally essentially bounded. Therefore, there exists a Filippov solution $x_i(t)$ that is absolutely continuous over $[0, \infty)$. Now, consider $t \in [t_i^s, t_i^{s+1}]$. The jump discontinuity of $e_{2,i}(t)$ at $t_i^{s+1}$ is given by $\Omega_i(t_i^{s+1}) = e_{2,i}(t_i^{s+1}) - \lim_{t \to (t_i^{s+1})^-} e_{2,i}(t)$. Therefore, the minimum dwell-time condition given by (12) can ensure that $\|e_{2,i}(t_i^s)\| \leq \|e_{2,i}(t_i^s)\| e^{-k_i(t_i^s-t_i^s)}$. It follows that the magnitude of the jump discontinuity is bounded by

$$
\left\|e_{2,i}(t_i^{s+1}) - \lim_{t \to (t_i^{s+1})^-} e_{2,i}(t)\right\| \leq V_T.
$$

Since $\|e_{2,i}(t)\|$ is strictly decreasing over $[t_i^s, t_i^{s+1}]$ by (16), then $\|e_{2,i}(t)\| \leq \|e_{2,i}(t_i^s)\|$ for all $t \in [t_i^s, t_i^{s+1}]$. The reset map in (2) may induce an instantaneous growth in (5) at $t_i^{s+1}$ where (17) implies $\|e_{2,i}(t_i^{s+1})\| \leq V_T + \|e_{2,i}(t_i^s)\| e^{-k_i(t_i^s-t_i^s)}$. Therefore, the minimum dwell-time condition given by (12) can ensure that $\|e_{2,i}(t_i^s)\| \leq \|e_{2,i}(t_i^{s+1})\|$, which is valid when $\|e_{2,i}(t_i^{s+1})\| > V_T > 0$. Observe that there exists some $t_i^s \in \mathbb{R}_{>0}$ such that $\|e_{2,i}(t_i^s)\| \leq V_T$. Provided the leader satisfies the maximum dwell-time condition in (11) for all $t \leq t_i^s$, then $\|C x_g - y_i(t_i^s)\| \leq \lambda_{\max}(C) \|e_{2,i}(t_i^s)\| + \lambda_{\max}(C) \|e_{i,1}(t_i^s)\| \leq 2 \lambda_{\max}(C) V_T$. Hence, by selecting $V_T \in \left(0, \frac{R_g}{2 \lambda_{\max}(C)}\right]$, it follows that $\|C x_g - y_i(t_i^s)\| \leq R_g$. Since $t_i^{s+1}$ and follower $i$ will be inside the feedback region after $t_i^{s+1}$. Moreover, $\|e_{1,i}(t)\| = 0$ and $\|e_{2,i}(t)\| = \|e_{2,i}(t_i^s)\| e^{-k_i(t_i^s-t_i^s)}$ for all $t \geq t_i^s$. Thus, $\|e_{2,i}(t)\| \to 0$ as $t \to \infty$. Since (13) does not have a restricted domain and is radically unbounded, then the stability result is global. \[\blacksquare\]
Remark 1: The proof of Theorem 2 formally excludes
Zeno behavior.
Remark 2: From Theorem 1 and Theorem 2, for stability and
consensus, for any \( i \) and \( s, \)
\[
\frac{1}{\lambda_{\text{max}}(A)} \ln \left( \frac{\lambda_{\text{max}}(A) V_T}{d_i^s} + 1 \right) \geq \frac{1}{k_i} \ln \left( \frac{\| e_{2,i}(t_i^s) \|}{\| e_{2,i}(t_{i+s}^s) - V_T \|} \right)
\]
(18)
With Theorem 1 and Theorem 2, we provide the following theorem
for achieving consensus of the followers.

Theorem 3: The states of the followers asymptotically
reach consensus to \( x_\beta \) if the maximum dwell-time condition
in (11) and the minimum dwell-time condition in (12) are
satisfied or all \( t_i^s \leq t_j^s \) \( (i \in F), \) and \( V_T \in \left(0, \frac{R^s}{\lambda_{\text{max}}(A)} \right). \)

Proof: Let \( i \in F. \) By Theorem 1, if the maximum
dwell-time condition in (11) is satisfied, then \( \| e_{2,i}(t) \| \leq V_T \)
for all \( t \geq 0. \) By Theorem 2, if the minimum dwell-time
condition in (12) is satisfied or all \( t_i^s \leq t_j^s \) \( (i \in F), \) then there exists a time \( T_i \in \mathbb{R}_>0 \) such that \( \| e_{2,i}(T_i) \| \leq V_T. \)
Therefore, \( \| Cx_q - y_i(T_i) \| \leq \lambda_{\text{max}}(C) \| e_{1,i}(T_i) \| + \lambda_{\text{max}}(C) \| e_{2,i}(T_i) \| \leq 2\lambda_{\text{max}}(C)V_T \)
\( \leq R_g \) as \( V_T \in \left(0, \frac{R^s}{\lambda_{\text{max}}(A)} \right). \) Then for \( t \geq T_i, \) follower \( i \) will be
inside the feedback region where \( \| e_{1,i}(t) \| = 0. \) Moreover, \( \| e_{2,i}(t) \| \rightarrow 0 \) as \( t \rightarrow \infty, \)
so \( \| e_{2,i}(t) \| = \| e_{1,i}(t) \| + \| e_{2,i}(t) \| \rightarrow 0 \) as \( t \rightarrow \infty. \)

IV. CONTROLLER SYNTHESIS WITH INTERMITTENT
COMMUNICATION AND MTL SPECIFICATIONS

In this section, we provide the framework and algorithms
for controller synthesis of the leader to satisfy the maximum
and minimal dwell-time conditions and the practical
constraints. The controller synthesis is conducted iteratively as
the state estimates for the followers are reset to the true state
values whenever they are serviced by the leader, and thus the
control inputs need to be recomputed with the reset values.

We assume that the communication is only possible at
discrete time instants, with \( T_i \) time periods apart and controlled
by the communication switching \( z_i(t). \) We define the discrete
time set \( T_d = \{ t[0], t[1], \ldots \}, \) where \( t[j] = jT_i, \) \( j \in \mathbb{N} \).
The maximum dwell-time \( \frac{1}{\lambda_{\text{max}}(A)} \ln \left( \frac{\lambda_{\text{max}}(A) V_T}{d_i^s} + 1 \right) \) in
(11) for robot \( i (i = 1, \ldots, Q), \) is in the interval \( [n_i T_i, (n_i + 1)T_i) \)
and the minimum dwell-time \( \frac{1}{k_i} \ln \left( \frac{\| e_{2,i}(t_i^s) \|}{\| e_{2,i}(t_j^s) - V_T \|} \right) \)
in (12) is in the interval \( [(m_i - 1)T_i, m_i T_i). \) We use the
following MTL specifications for encoding the maximum
dwell-time condition and the minimum dwell-time condition
\( \eta \in (0, R) \) is a user-defined parameter):
\[
\phi_1 = \bigwedge_{1 \leq i \leq Q} \left( \square \diamond [0, n_i) \| y_0 - \bar{y}_i \| \leq \eta \right),
\]
\[
\phi_2 = \bigwedge_{1 \leq i \leq Q} \left( \square \left( \| y_0 - \bar{y}_i \| \leq \eta \Rightarrow \square [1, m_i) \| y_0 - \bar{y}_i \| > \eta \right) \right),
\]
(19)
where \( \phi_1 \) means “for any follower \( i, \) the leader needs to be
within \( \eta \) distance from the estimated position of the follower
\( i \) at least once in any \( n_i T_i \) time periods”, and \( \phi_2 \) means
“each time the leader is within \( \eta \) distance from the estimated
position of the follower \( i, \) it should not be within \( \eta \) distance
from the estimated position of the follower \( i \) again for the
next \( m_i T_i \) time periods”.

The leader also needs to satisfy an MTL specification \( \phi_p \)
for the practical constraints. One example of \( \phi_p \) is as follows:
\[
\phi_p = \square \diamond \left( \{ y_0 \in G_1 \} \lor \{ y_0 \in G_2 \} \right) \land \square \diamond (y_0 \in D). \]
(20)
which means “the leader robot needs to reach the charging
station \( G_1 \) or \( G_2 \) at least once in any \( cT_i \) time periods, and
it should always remain in the region \( D \).”
Combining \( \phi_1, \phi_2 \) and \( \phi_p, \) the MTL specification for the
leader is \( \phi = \phi_1 \land \phi_2 \land \phi_p. \)

We use \( \phi^j_{\ell} \) to denote the formula modified from the MTL
formula \( \phi \) when \( \phi \) is evaluated at time index \( j \) and the current
time index is \( \ell. \) \( \phi^j_{\ell} \) can be calculated recursively as follows
(we use \( \pi_j \) to denote the atomic predicate \( \pi \) evaluated at
time index \( j \)):
\[
[\phi^j_{\ell}] = \begin{cases} 
\pi_j, & \text{if } j > \ell \\
\top, & \text{if } j \leq \ell \text{ and } y_j^i \in O(\pi) \\
\bot, & \text{if } j \leq \ell \text{ and } y_j^i \notin O(\pi)
\end{cases}
\]
(21)
\[
\phi_1 \land \phi_2 \land \phi^j_{\ell} = \bigwedge_{j' \in [j+1, \ell]} \left( [\phi_2]_{j'} \land \bigwedge_{j \leq j' < j'} [\phi_1]_{j'} \right).
\]
If the MTL formula \( \phi \) is evaluated at the initial time index
(which is the usual case when the task starts at the initial
time), then the modified formula is \( [\phi]^j_0. \)

Algorithm 1 shows the controller synthesis approach with
intermittent communication and MTL specifications. The
controller synthesis problem can be formulated as a sequence
of mixed integer linear programming (MILP) problems,
denoted as MILP-sol in Line 3 and expressed as follows:
\[
\text{arg min } J(u^0_0) = \| u^0_0 \|
\]
(22)
such that: \( x^j+1 = A_0 x^j + B^0 u^j, \)
\[
\forall i = 1, \ldots, Q, \forall j = \ell, \ldots, \ell + N - 1,
\]
(23)
\[
\hat{x}^j+1 = A^0 \hat{x}^j + B^0 u^j,
\]
\[
\forall i = 1, \ldots, Q, \forall j = \ell, \ldots, \ell + N - 1,
\]
(24)
\[
u_{0,\text{min}} \leq u^0_0 \leq u_{0,\text{max}}, \forall i = 1, \ldots, Q,
\]
\[
\forall j = \ell, \ldots, \ell + N,
\]
(25)
\[
(y_{\ell+1}^\ell, 0) \models [\phi^\ell_0, \phi^j_0]
\]
(26)
where the time index \( \ell \) is initially set as \( 0, N \in \mathbb{Z}_{>0} \)
is the number of time instants in the control horizon,
\[
(y_{\ell+1}^\ell, 1, \ldots, y_{\ell+1}^\ell, \ldots, y_{\ell+1}^\ell, (\sqrt{0} - 1, 0) \text{ is the control input signal of the}
\]
leader, the input values are constrained to \( [u_{0,\text{min}}, u_{0,\text{max}}], \)
\( A^0, B^0, C^0, A, B \) and \( C \) are converted from \( A^0, B^0, C^0, \)
\( A, B \) and \( C \) respectively for the discrete-time state-space
representation, and \( u^i_0 \) are follower control inputs from (6).
Note that we only require the trajectory \( y_{(\ell+1)}^\ell \) to weakly
Algorithm 1 Controller synthesis of MASs with intermittent communication and MTL specifications.

1: Inputs: $x_0$, $k_i$
2: $\ell \leftarrow 0$
3: Solve MILP-sol to obtain the optimal inputs $u^{q+\ell}_i$ ($q = 0, 1, \ldots, N - 1$)
4: while $|C x_2 - y_i(t(\ell))| > R_g$ for some $i$ do
5: $W = \{i \mid \|y_i - y_i(t(\ell))\| \leq \eta\}$
6: if $|W| \neq 0$ then
7: $\forall i \in W$, update $\hat{x}_i^{t}$ in constraint (24) and change constraint (24) as follows:
8: $\hat{x}_i^{t+1} = Ax_i^i_t + Bu_i^i_t, \forall i = 1, \ldots, Q,$
9: $\forall j = \ell, \ell + 1, \ldots, \ell + N - 1,$
10: $\hat{x}_i^t = x_i^t, \forall i \in W$.
11: end if
12: end while
13: Return $u^{*} = (u_0^{q_0}, u_0^{q_1}, \ldots)$

satisfy $\phi$ as $\ell + N - 1$ may be less than the necessary length $\|\phi\|$.

At each time index $\ell$, we check if there exists any follower that is being serviced (Line 5). If there are such followers, we update the state estimates of those followers with their true state values (Line 7). Then we modify the MTL formula as in (21) and the updated $m_i$ (Line 8). The MILP is solved for time $\ell$ with the updated state values and the modified MTL formula $|\phi_G^0|$ (Line 9). The previously computed leader control inputs are replaced by the newly computed control inputs from time index $\ell$ to $\ell + N - 1$ (Line 10).

We use $\hat{t}_i^{t+1}$ to denote the $(s + 1)^{th}$ time that $\|y_i(t) - y_i(t_i)\| \leq \eta$ holds in the discrete time set $T_d$ for follower $i$, i.e.,

$$\hat{t}_i^{t+1} \triangleq \inf \{ t \geq \hat{t}_i^t : (t \in T_d) \land (\|y_i(t) - y_i(t_i)\| \leq \eta) \}.$$  

We design the communication switching signal $\zeta_i$ as follows:

$$\zeta_i(t) = \begin{cases} 1, & \text{if } t = \hat{t}_i^s \text{ for some } s; \\ 0, & \text{otherwise.} \end{cases} \quad (27)$$

Finally, we present Theorem 4 which provides theoretical guarantees for achieving correctness, stability and consensus (in Problem 1).

**Theorem 4:** With the observers in (2), follower controllers in (6), communication switching signal in (27), if each optimization is feasible in Algorithm 1 and $V_T \in \left(0, \min \left(\frac{R_g}{2\max(C)}, \frac{R_g}{2\max(C)} \right) \right)$ where $\eta \in [0, R_g)$, then Algorithm 1 terminates within finite time, with the MTL specification $\phi$ weakly satisfied and the followers asymptotically reaching consensus to the state $x_g$.

**Proof:** We first use induction to prove that $\hat{t}_i^{t_0} = t_0^i$ holds for each $i$ and $s$. For each $i$, if $s = 0$, then $\hat{t}_i^{t_0} = t_0^i = 0$. Now assume that $\hat{t}_i^{s-1} = t_0^i$ holds and we prove that $\hat{t}_i^{s+1} = t_0^{s+1}$ holds. If each optimization is feasible in Algorithm 1, then $\hat{t}_i^{s+1} - \hat{t}_i^{s} = \hat{t}_i^{s+1} - t_0^{s+1} \leq n_i T_s \leq \frac{1}{\lambda_{max}(A)V_T} + 1$. Then, following the analysis in the proof of Theorem 1, we can derive that $\|e_i(t_0^{s+1})\| \leq V_T$. Thus, we have $\|y_i(t_0^{s+1}) - y_0(t_0^{s+1})\| \leq \|Cx_i(t_0^{s+1}) - Cx_0(t_0^{s+1})\| \leq \|\lambda_{max}(C)\| V_T + \eta$. Therefore, if $V_T \leq \frac{R_g}{\lambda_{max}(C)}$, then $\|y_i(t_0^{s+1}) - y_0(t_0^{s+1})\| \leq R_g$. According to the communication switching signals in (27), we have $\zeta_i(t_0^{s+1}) = 1$. Thus, from the definition of $t_0^{s+1}$ in Section II-B, we have $t_0^{s+1} = t_0^{s+1}$ holds. Therefore, we have proven through induction that $\hat{t}_i^{t_0} = t_0^i$ hold for each $i$ and $s$.

If each optimization is feasible in Algorithm 1, then the MTL specification $\phi$ is weakly satisfied. With $\hat{t}_i^{t_0}$, the maximum dwell-time condition in (11) and the minimum dwell-time condition in (12) are satisfied or all $t_0^i \leq t^s_i$ ($i \in F$). From Theorem 3, if $V_T \in \left(0, \frac{R_g}{2\max(C)} \right)$, then for each $i \in F$, there exists a time $T$ such that follower $i$ will be inside the feedback region for $t \geq T$. Thus, at time $t = \max_{i \in F} T_i$, $|C x_2 - y_i(t)| \leq R_g$ holds for any $i \in F$, i.e., Algorithm 1 is guaranteed to terminate within finite time. Finally, if $V_T \in \left(0, \min \left(\frac{R_g}{2\max(C)}, \frac{R_g}{2\max(C)} \right) \right)$, then the followers asymptotically reach consensus to $x_g$.

**V. IMPLEMENTATION**

We now demonstrate the controller synthesis approach on the example in Fig. 1 (in Section I). The leader is a quadrotor modeled as a three dimensional six degrees of freedom (6-DOF) rigid body [23]. We denote the system state as $x_0 = [p_0, q_0, \theta_0, \Omega_0]^T \in \mathbb{R}^{12}$, where $p_0 = [x_0, y_0, z_0]^T$ and $\tilde{p}_q = [\tilde{x}_q, \tilde{y}_q, \tilde{z}_q]^T$ are the position and velocity vectors of the quadrotor. The vector $\theta_0 = [\alpha_0, \beta_0, \gamma_0]^T \in \mathbb{R}^3$ includes the roll, pitch and yaw Euler angles of the quadrotor. The vector $\Omega_0 \in \mathbb{R}^3$ includes the angular velocities rotating around its body frame axes. The general nonlinear dynamic model of such a quadrotor is given by

$$m_q \ddot{q}_q = r(\theta_0)T_q \mathbf{e}_3 - m g \mathbf{e}_3,$$

$$\dot{\theta}_0 = H(\theta_0) \Omega_0,$$

$$\dot{\Omega}_0 = -\Omega_0 \times \Omega_0 + \tau_q,$$

where $m_q$ is the mass, $g$ is the gravitational acceleration, $I$ is the inertia matrix, $r(\theta_0)$ is the rotation matrix representing the body frame with respect to the inertia frame (which is a function of the Euler angles), $H(\theta_0)$ is the nonlinear mapping matrix that projects the angular velocity $\Omega_0$ to the Euler angle rate $\dot{\theta}_0$, $\mathbf{e}_3 = [0, 0, 1]^T$, $T_q$ is the thrust of the quadrotor, and $\tau_q \in \mathbb{R}^3$ is the torque on the three axes. The control input is $u_0 = [u_{0,1}, u_{0,2}, u_{0,3}, u_{0,4}]^T$, where $u_{0,1}$ is the vertical velocity command, $u_{0,2}$, $u_{0,3}$ and $u_{0,4}$ are the angular velocity commands around its three body axes. The input values $u_{0,1}$, $u_{0,2}$, $u_{0,3}$ and $u_{0,4}$ are all bounded by $[-100, 100]$. By adopting the small-angle assumption and
then linearizing the dynamic model around the hover state, a linear kinematic model can be obtained as follows:
\[
\dot{x}_0 = A_0 x + B_0 u_0, \tag{29}
\]
where \( x_0 = [x_{q,1}, x_{q,2}, x_{q,3}, \dot{x}_{q,1}, \dot{x}_{q,2}, \alpha_q, \beta_q, \gamma_q]^T \) is the state of the kinematic model of the quadrotor (leader), \( A_0 \in \mathbb{R}^{8 \times 8} \) and \( B_0 \in \mathbb{R}^{8 \times 4} \). For the 3-D position representation, \( y_0 = [x_{q,1}, x_{q,2}, x_{q,3}]^T \).

We use the following simplified dynamics for the followers
\[
\dot{x}_{i,1} = x_{i,1} + u_{i,1} + d_{i,1},
\dot{x}_{i,2} = x_{i,2} + u_{i,2} + d_{i,2},
\dot{x}_{i,3} = 0,
\] (30)
where \( x_{i,1}, x_{i,2} \) and \( x_{i,3} \) are the 3-D positions of follower \( i \). Note that the vertical positions of the followers are constant.

For the state space representation, \( x_i = [x_{i,1}, x_{i,2}, x_{i,3}]^T \), \( u_i = [u_{i,1}, u_{i,2}]^T \), \( d_i = [d_{i,1}, d_{i,2}]^T \) and \( y_i = [x_{i,1}, x_{i,2}, x_{i,3}]^T \). The initial 3-D positions of the three followers are \([-20, -20, 0]^T, [20, 30, 0]^T \) and \([40, -40, 0]^T \), respectively. The initial 3-D position of the leader is \([-5, -30, 5]^T \). The consensus state \( x_g \) is set as \([0, 0, 0]^T \).

The random disturbances \( d_i \) are bounded, i.e., \( \|d_i(t)\| \leq \bar{d}_i \), where \( \bar{d}_1 = 0.04, \bar{d}_2 = 0.03 \) and \( \bar{d}_3 = 0.02 \).

For consensus, we consider the following control law from (6):
\[
u_i(t) = -\dot{x}_i + k_1 e_{2,i}(t),
\]
where \( \dot{x}_i \) is the estimate of \( x_i \), \( k_1 = 0.1, k_2 = 0.15 \) and \( k_3 = 0.2 \), respectively.

We consider two different scenarios with two different MTL specifications for the practical constraints.

**Scenario 1:**
The leader needs to reach the charging station \( G_1 \) or \( G_2 \) at least once in any \( 6T \) time, and it should always remain in region \( D \), where the two charging stations \( G_1 \) and \( G_2 \) are rectangular cuboids with length, width and height being 2, 2 and 5, centered at \([-20, 10, 2.5]^T \) and \([25, 0, 2.5]^T \), respectively. The region \( D \) is a rectangular cuboid centered at \([0, 0, 7]^T \) with length, width and height being 30, 30 and 6, respectively (see Fig. 1). This specification is expressed as
\[
\phi^1_p = \Box \Diamond_{[0,6]} ((y_0 \in G_1) \lor (y_0 \in G_2)) \land \Box (y_0 \in D).
\]

**Scenario 2:**
The leader needs to reach the charging station \( G_1 \) or \( G_2 \) at least once in any \( 6T \) time, always remain in region \( D \), and never stay in region \( E \) for more than \( 2T \) time, where the region \( E \) is a rectangular cuboid centered at \([0, 0, 6]^T \) with length, width and height being 15, 15 and 4, respectively. This specification is expressed as
\[
\phi^2_p = \Box \Diamond_{[0,6]} ((y_0 \in G_1) \lor (y_0 \in G_2)) \land \Box (y_0 \in D)
\]
\[
\land \neg \Diamond \Box_{[0,2]} (y_0 \in E). \tag{31}
\]

We set \( R_g = R = 5 \), \( V_T = 1 \), \( \eta = 4 \), \( T_s = 0.5 \) and \( N = 20 \). Fig. 2 shows the simulation results in Scenario 1. The obtained input signals as shown in Fig. 2 (a) gradually decrease as the followers approach \( R_g \). Fig. 2 (b) shows the 2-D planar plot of the trajectories of three followers and a leader. \( \|e_{1,i}(t)\| \) in Scenario 2 (c) is uniformly bounded by \( V_T = 1 \). \( \|e_{1,2}(t)\| \) in Scenario 2 (d) is monotonically decreasing when the followers approach consensus to \( x_g \).

**VI. Conclusion**

We presented a metric temporal logic approach for the controller synthesis of a multi-agent system (MAS) with intermittent communication. We iteratively solved a sequence of mixed-integer linear programming problems for provably achieving the correctness, stability of the switched system and consensus of the followers. Future work will also extend the implementations to more realistic dynamic models for the followers and experiments on the hardware testbed.

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Fig. 3: Results with MTL specification $\phi_p^2$ for the practical constraints: (a) the obtained optimal input signals; (b) 2-D planar plot of the trajectories of three followers and a leader; (c) $\|e_{1,1}(t)\|$; (d) $\|e_{1,2}(t)\|$.

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